Government Engineering College Jhalawar
Department of Management Studies
MBA I Year(OR)

Attempt Any four question **Date:06/03/2018** **MM:10**

Q.1. two players R and C have one coin each. After a signal each of them exposes the coin . Player R wins a unit when there are two heads, wins nothing when there are two tails and loose ½ unit when there is one head and 1 tail. Determine the payoff matrix, the best strategies for each player and the value of game.

when there ...
the Pay-off matrix, the best ...

Solution : The Pay-off Matrix of the game ...

$$R \begin{matrix} \\ H \\ T \end{matrix} \overset{\begin{matrix} & C & \\ H & & T \end{matrix}}{\begin{pmatrix} 1 & -1/2 \\ -1/2 & 0 \end{pmatrix}}$$

As there is no saddle point, we will solve it by using oddoment method

R	C: H	C: T	Row Oddoments
H	1	−1/2	1/2
T	−1/2	0	3/2
Column Oddoments	1/2	3/2	

R will play in the ratio : $\frac{1}{2} : \frac{3}{2}$; C will play in the ratio : $\frac{1}{2} : \frac{3}{2}$

Optimal Strategy for R : $\begin{pmatrix} \frac{1}{4} \\ \frac{3}{4} \end{pmatrix}$; Optimal Strategy for C : $\begin{pmatrix} \frac{1}{4} & \frac{3}{4} \end{pmatrix}$

$$\text{Value of the game} = \frac{1 \times \frac{1}{2} - \frac{1}{2} \times \frac{3}{2}}{2} = -\frac{1}{8}$$

Example 12 : A game has Pay-off Matrix

Q.2. A and B play a game in which each has three coins, a 5 paise, a 10 paise and a 20 paise coin. Each player selects a coin without the knowledge of other's choice. If the sum of the coins is an odd amount, A wins B's coin and if the sum is even , B wins A's coin. Find the best strategy for each player and value of game.

game.

Solution : The game matrix will be :

			'B'		
			B_1	B_2	B_3
			5	10	20
	A_1	5	-5	10	20
'A'	A_2	10	5	-10	-10
	A_3	20	5	-20	-20

Applying law of dominance we can delete 3rd row and 3rd column. The reduced ame matrix is

		'B'		
		B_1	B_2	
'A'	A_1	-5	10	15
	A_2	5	-10	15
		10	20	

Theory

On interchanging the differences we get :

A_1	-5	10	15
A_2	5	-10	15
	20	10	

$$\text{Optimal Strategy for A : } \begin{pmatrix} \frac{15}{30} \\ \frac{15}{30} \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$$

$$\text{Optimal Strategy for B : } \begin{pmatrix} \frac{20}{30} & \frac{10}{30} & 0 \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

$$\text{Value of the game} = \frac{-5\times15+5\times15}{30} = 0$$

The game is fair for both the players.

Q.3. Find the solution of following game by using law of dominance:

		B			
		B_1	B_2	B_3	B_4
A	A_1	3	2	4	0
	A_2	3	4	2	4
	A_3	4	2	4	0
	A_4	0	4	0	8

(M.D.U. Ajmer, 1996)

Solution : Comparing the strategies A_3 and A_1 of A we see that A_3 dominates A_1. The reduced matrix is

'B'

'A'	B_1	B_2	B_3	B_4
A_2	3	4	2	4
A_3	4	2	4	0
A_4	0	4	0	8

Again comparing B_1 and B_3 for B, we find that B_3 dominantes B_1. The reduced me matrix is

	B_2	B_3	B_4
A_2	4	2	4
A_3	2	4	0
A_4	4	0	8

Again comparing B_2 with the average of B_3 and B_4 $\begin{pmatrix}3\\2\\4\end{pmatrix}$ we find that B_2 can be deleted from the matrix. The reduced matrix is

	B_3	B_4
A_2	2	4
A_3	4	0
A_4	0	8

Now we find that on comparing A_1 with the average of A_3 and A_4 (2, 4), A_2 can be deleted. The reduced game matrix is

$$\begin{array}{c} \\ A_3 \\ A_4 \end{array} \begin{array}{c} B_3 \quad B_4 \\ \begin{pmatrix} 4 & 0 \\ 0 & 8 \end{pmatrix} \end{array}$$

After finding the differences, of the pay-offs of rows and columns and interchanging them we get

$$\begin{array}{c} \\ A_3 \\ A_4 \\ \\ \end{array} \begin{array}{c} B_3 \quad B_4 \\ \begin{pmatrix} 4 & 0 \\ 0 & 8 \end{pmatrix} \begin{array}{c} 8 \\ 4 \end{array} \\ 8 \quad\; 4 \end{array}$$

Optimal strategy for A : $\begin{pmatrix} 0 \\ 0 \\ \frac{2}{3} \\ \frac{1}{3} \end{pmatrix}$ Optimal Strategy for B : $\left(0, 0, \frac{2}{3}, \frac{1}{3}\right)$

Value of the game $= \dfrac{4\times 8 + 0 \times 4}{12} = \dfrac{32}{12}$ or $\dfrac{8}{3}$

Example 16 : Solve the followi...

Q.4.Solve the game whose payoff matrix is given below:

A		B	
		B_1	B_2
	A_1	6	-1
	A_2	0	4
	A_3	4	3

Solution : The given game is a 3×2 game. The possible 2×2 sub-games are

$$\text{I}\quad \begin{array}{c} \\ A_1 \\ A_2 \end{array}\begin{array}{c} B_1 \quad B_2 \\ \begin{bmatrix} 6 & -1 \\ 0 & 4 \end{bmatrix} \end{array}\begin{array}{c} \\ 4 \\ 7 \end{array} \qquad \text{II}\quad \begin{array}{c} \\ A_1 \\ A_3 \end{array}\begin{array}{c} B_1 \quad B_2 \\ \begin{bmatrix} 6 & -1 \\ 4 & 3 \end{bmatrix} \end{array} \qquad \text{III}\quad \begin{array}{c} \\ A_1 \\ A_3 \end{array}\begin{array}{c} B_1 \quad B_2 \\ \begin{bmatrix} 0 & 4 \\ 4 & 3 \end{bmatrix} \end{array}$$

The value of game I is $\frac{24}{11}$, value of second game is 3 and value of III game is $\frac{16}{5}$. Player A is a maximizing player; he will select that sub-game whose game value is maximum. Thus, Player A will select III sub-game with largest game value $\frac{16}{5}$. The optimal strategies for original game will be as follows :

Optimal Strategy for player A $\begin{pmatrix} 0 \\ \frac{1}{5} \\ \frac{4}{5} \end{pmatrix}$; Optimal Strategy for player B $\left(\frac{1}{5}, \frac{4}{5}\right)$

The value of the game is $\frac{16}{5}$

Q.5. A has two ammunition stores, one of which is twice as valuable as the other. B is an attacker who can destroy an undefended store but he can attack only any one of them at a time. A knows that B is about to attack one of the stores but does not know which one. What should he do? Note that he can successfully defend one store at a time?

$$\begin{array}{ccccc} & & \multicolumn{2}{c}{B} & \\ & & \text{Attack I} & \text{Attack II} & \\ A & \begin{array}{r}\text{Defend I} \\ \text{Defend II}\end{array} & \multicolumn{2}{c}{\begin{pmatrix} 0 & -1 \\ -2 & 0 \end{pmatrix}} & \begin{array}{c}1 \\ 2\end{array} \\ & & 2 & 1 & \end{array}$$

Row differences

Column differences

Step II Interchanging the differences we get $\begin{pmatrix} 0 & -1 \\ -2 & 0 \end{pmatrix}\begin{matrix} 2 \\ 1 \end{matrix}$
$\quad\quad 1 \quad 2$

Optimal Strategy for A : $\begin{pmatrix} \frac{2}{2+1} \\ \frac{1}{2+1} \end{pmatrix} = \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix}$

Optimal Strategy for B : $\begin{pmatrix} \frac{1}{1+2} & \frac{2}{1+2} \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

Value of the game $= \dfrac{0 \times 2 + (-2) \times 1}{3} = \dfrac{-2}{3}$