Subject → Dynamics of Machines

Code → 5ME2A

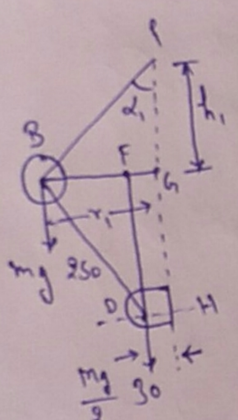
Question Model Paper

Pradeep Suman

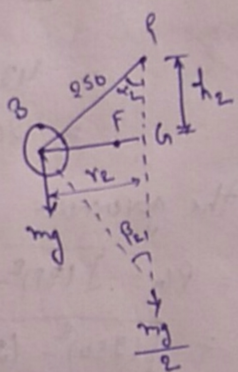
Mechanical Department

---

① A porter governor has all four arms 250 mm long. The upper arm are attached on the axis of rotation and the lower arm are attached to the sleeve at a distance of 30 mm from the axis. The mass of each ball is 5 Kg and the sleeve has a mass of 50 Kg. The external radii of rotation are 150 mm and 200 mm. Determine the range of speed of the governor.



(minimum position)



(Maximum position)

Let N_1 = Minimum speed when $r_1 = BG = 150$ mm

N_2 = Maximum speed when $r_2 = BG = 200$ mm

$$h_1 = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(250)^2 - (150)^2}$$

$$= 200 \text{ mm}$$

$$= 0.2 \text{ m}$$

height of the governor -

$h_1 = PG = \sqrt{(BP)^2 - (BG)^2}$

$= \sqrt{(250)^2 - (150)^2} = 0.2\,m$

$BF = BG - FG = 150 - 30 = 120\ mm \quad (\because FG = DH)$

$DF = \sqrt{DB^2 - BF^2}$

$= \sqrt{(250)^2 - (120)^2} = 219\ mm$

$\tan\alpha_1 = BG/PG = 150/200 = 0.75$

$\tan\beta_1 = BF/DF = 120/219 = 0.548$

$q_1 = \tan\beta_1/\tan\alpha_1 = \frac{0.548}{0.75} = 0.731$

$\because$ We know that -

$$(N_1)^2 = \frac{m + \frac{M}{2}(1+q_1)}{m} \times \frac{895}{h_1}$$

$$= \frac{5 + \frac{50}{2}(1+0.731)}{5} \times \frac{895}{0.2}$$

$$= 43206$$

$$= 208\ r.p.m.$$

Again height of the governor -

$h_2 = PG = \sqrt{(BP)^2 - (BG)^2}$

$= \sqrt{(250)^2 - (200)^2}$

$= 150\ mm$

$= 0.15\ m$

$BF = BG - FG = 200 - 30 = 170\ mm$

$DF = \sqrt{(DB)^2 - (BF)^2}$

$= \sqrt{(250)^2 - (170)^2}$

$= 183\ mm$

$\tan \alpha_2 = BG/PG = 200/150 = 1.333$

$\tan \beta_2 = BF/DF = 170/183 = 0.93$

$q_2 = \tan \beta_2 / \tan \alpha_2 = \frac{0.93}{1.333} = 0.7$

$$\therefore (N_2)^2 = \frac{m + \frac{M}{2}(1+q_2)}{m} \times \frac{895}{h_2}$$

$$= \frac{5 + \frac{50}{2}(1+0.7)}{5} \times \frac{895}{0.15}$$

$$= 56683$$

$$\Rightarrow N_2 = 238 \text{ R.p.m.}$$

We know that range of speed $= N_2 - N_1$

$= 238 - 208$

$= 30$ R.p.m. Ans

(2) The number of teeth on each of the two equal gears in mesh are 40. The teeth have 20° involute profile and the module is 6 mm. If the arc of contact is 1.75 times the circular pitch, find the addendum.

$\therefore$ pitch of Circular -

$p_c = \pi m = \pi \times 6 = 18.85$ mm

$\therefore$ Length of arc of Contact -

$= 1.7 p_c = 1.75 \times 18.85$

$= 33$ mm

Length of path of Contact = length of arc of contact $\times \cos\phi$

$= 33 \cos 20°$

$= 31$ mm

$R_A = r_A =$ Radius of the addendum circle of each wheel

∵ We know that pitch circular radii of each wheel -

$$R = r = \frac{mT}{2}$$

$$= \frac{6 \times 40}{2} = 120 \text{ mm}$$

and length of path of contact -

$$31 = \sqrt{(R_A)^2 - R^2 \cos^2\phi} + \sqrt{(r_A)^2 - r^2 \cos^2\phi} - (R+r)\sin\phi$$

$$= 2\left[\sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi\right]$$

$$\frac{31}{2} = \sqrt{(R_A)^2 - (120)^2\cos^2 20^\circ} - 120\sin 20^\circ$$

$$15.5 = \sqrt{(R_A)^2 - 12715} - 41$$

$$(15.5 + 41)^2 = (R_A)^2 - 12715$$

$$R_A = 126.12 \text{ mm}$$

∵ We know that the addendum of the wheel -

$$= R_A - R$$

$$= 126.12 - 120$$

$$= 6.12 \text{ Ans}$$

③ Determine the minimum number of teeth required on a pinion, in order to avoid interference which is to gear with -

ⓐ a wheel to give a gear ratio of 3 to 1

ⓑ an equal wheel.

The pressure angle is 20° and a standard addendum of 1 module for the wheel may be assumed.

(a) Minimum number of teeth for a gear ratio of 3:1 -

∵ We know that minimum number of teeth required on a pinion -

$$t = \frac{2 \times A_w}{G\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1\right]}$$

$$t = \frac{2 \times 1}{3\left[\sqrt{1+\frac{1}{3}\left(\frac{1}{3}+2\right)\sin^2 20°} - 1\right.}$$

$$= \frac{2}{0.133} = 15.04 \text{ OR } 16 \text{ Ans}$$

(b) Minimum number of teeth for equal wheel -

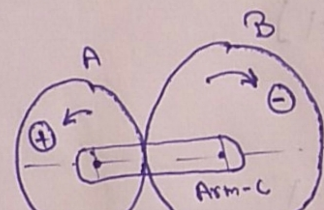
$$\because t = \frac{2 \times A_w}{\sqrt{1+3\sin^2\phi} - 1}$$

$$= \frac{2 \times 1}{\sqrt{1+3\sin^2 20°} - 1}$$

$$= \frac{2}{0.162}$$

$$= 12.34 \text{ OR } 13 \text{ Ans}$$

(4) In an epicyclic gear train, an arm carries two gears - A and B having 36 and 45 teeth respectively. If the arm rotates at 150 R.P.m. in the anticlockwise direction about the centre of the gear A which is fixed determine the speed of gear B. If the gear-A instead of being fixed, makes 300 r.p.m. in the clockwise direction, what will be the speed of gear - B?



∵ Given: $T_A = 36$ $T_B = 45$

$N_C = 150$ r.p.m.

Step No.	Condition of motion	Revolution of elements		
		Arm-C	Gear-A	Gear-B
1.	Arm fixed - gear A rotates through +1 revolution	0	+1	$-T_A/T_B$
2.	Arm fixed - gear A rotates through +x revolution	0	+x	$-x\left(\frac{T_A}{T_B}\right)$
3.	Add +y revolution	+y	x+y	+y
4.	Total Motion	+y	x+y	$y-x\left(\frac{T_A}{T_B}\right)$

∴ Speed of gear B, when gear-A is fixed -

Since the speed of arm is 150 r.p.m. anticlockwise therefore from the fourth row of the table

$y = +150$ rpm

Also the gear A is fixed, therefore -

$x+y = 0$

$x = -y = -150$ r.p.m.

∵ Speed of gear-B

$$N_B = y - x\left(\frac{T_A}{T_B}\right)$$

$$= 150 + 150\left(\frac{36}{45}\right)$$

$$= +270 \text{ r.p.m.}$$

$$N_B = 270 \text{ r.p.m. (Anticlockwise)}$$ Ans

Speed of gear-B, when gear-A makes 300 r.p.m. clockwise-

∵ Since the gear-A makes 300 r.p.m. clockwise therefore from the 4^{th} row of the table-

$$x + y = -300$$

$$x = -300 - y$$

$$x = -300 - 150$$

$$x = -450 \text{ R.P.M.}$$

∵ Speed of gear-B-

$$N_B = y - x\left(\frac{T_A}{T_B}\right)$$

$$N_B = 150 + 450\left(\frac{36}{45}\right)$$

$$N_B = +510 \text{ r.p.m (Anticlockwise)}$$ Ans

5. 4 masses of m_1, m_2, m_3 and m_4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of the rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between succesive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if the radius of rotation is 0.2 m.

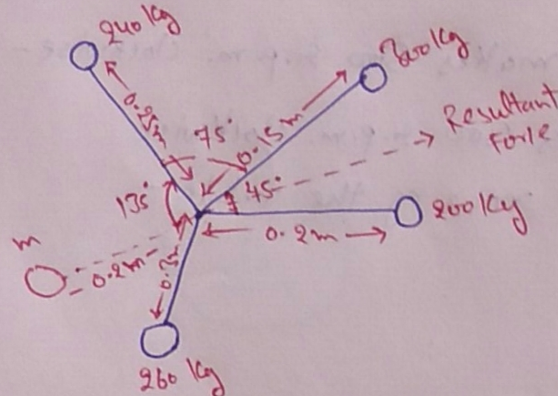
∴ Given - $m_1 = 200$ Kg $r_1 = 0.2$ m $\theta_1 = 0°$ $r = 0.2$ m

$m_2 = 300$ Kg $r_2 = 0.15$ m $\theta_2 = 45°$

$m_3 = 240$ Kg $r_3 = 0.25$ m $\theta_3 = 45° + 75° = 120°$

$m_4 = 260$ Kg $r_4 = 0.30$ m $\theta_4 = 45° + 75° + 135° = 255°$

(a) First of all draw the space diagram showing the position of all the given masses as show data.



(b) Since the centrifugal force of each mass is proportional to the product of the mass and radius -

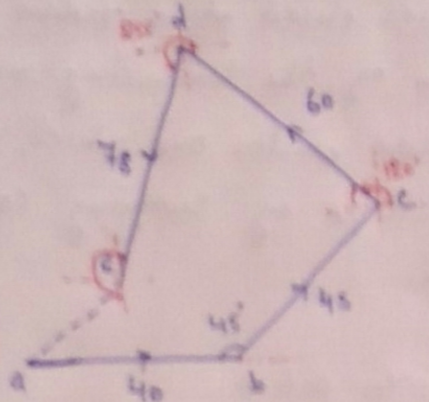
$m_1 r_1 = 200 \times 0.2 = 40$ Kg.m

$m_2 r_2 = 300 \times 0.15 = 45$ Kg.m

$m_3 r_3 = 240 \times 0.25 = 60$ Kg.m

$m_4 r_4 = 260 \times 0.30 = 78$ Kg.m

(c) Draw the vector diagram with the above value, to some Suitable scale, as shown figure. The closing side of the polygon are represent the resultant force. By measurement, we found that 'ae' = 23 Kg.m



(a) The balancing force is equal to the resultant force, but opposite in direction as shown in figure. Since the balancing force is proportional to m.r -

$m \cdot r$ = Vector ea

$m \cdot 0.2 = 23$ Kg. m

$m = 23/0.2 = 115$ Kg Ans

By measurement we also find that the angle of inclination of the blancing mass (m) from the horizontal mass of 200 kg

$\theta = 201°$ Ans

(5) An inside cylinder of Locomotive has its cylinder centre line 0.7 m appart and has a stroke of 0.6 m. The rotating masses per cylinder are equivalant to 150 Kg at the crank pin and the reciprocating masses per cylinder to 180 Kg. The wheel centre lines are 1.5 m appart. The cranks are at right angle.

The whole of the rotating and 2/3 of the reciprocating

masses are to be balanced by masses placed at a radius of 0.6 m
Find the magnitude and direction of the balancing masses.
Find the fluctuation in rail pressure under one wheel, variation of Tractive effort and the magnitude of swaying couple at a crank speed of 300 r.p.m.

∵ Given-

$a = 0.7 m$ $\quad m_1 = 150$ Kg $\quad r_A = r_D = 0.6$ m

$l_B = l_C = 0.6 m$ $\quad m_2 = 180$ Kg $\quad N = 300$ rpm

$r_B = r_C = 0.3 m$ $\quad c = 2/3$ $\quad \omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60} = 31.42$ rad/s

$$\therefore m = m_B = m_C = m_1 + c m_2$$
$$= 150 + \frac{2}{3} \times 180$$
$$= 270 \text{ Kg}$$

(a) magnitude and direction of the balancing masses-

Let m_A & m_D = magnitude of the balancing masses

θ_A & θ_D = Angular position of the balancing masses m_A and m_D from the crank

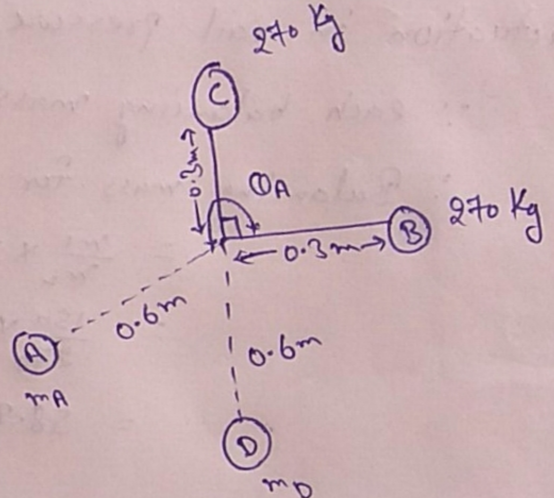
(b) First of all draw the space digram to show the position of the planes, of the wheel and the cylinder. Since the crank of the cylinder are at right angle therefor assuming the position of crank of the cylinder - B in the horizontal-

Plane	mass (m) Kg	Radius (r) m	Centrifugal force $\div \omega^2$ (m.r) Kg.m	Distance from Plane (A) (l) m	Couple $\div \omega^2$ (m.r.l) Kg.m^2
A (R.P.)	m_A	0.6	$0.6\, m_A$	0	0
B	270	0.3	81	0.4	32.4
C	270	0.3	81	1.1	89.1
D	m_D	0.6	$0.6\, m_D$	1.5	$0.9\, m_D$

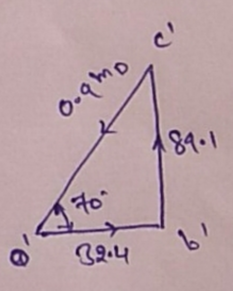
Ⓒ Now draw the couple polygon from the given data in table.

$$0.9\, m_D = \text{vector } c'o'$$
$$= 94.5 \text{ Kg-m}^2$$
$$m_D = 105 \text{ Kg} \quad \text{Ans}$$

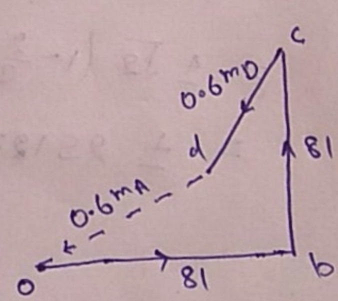
Ⓐ position of planes



Ⓑ Angular position of masses



Ⓒ Couple polygon



Ⓓ Force polygon

(d) Angular position of the balancing mass - D

$\theta_D = 250^\circ$ Ans

(e) Balancing mass A -

$0.6\, m_A$ = vector do

$0.6 m_A = 63$ Kg.m

$m_A = 105$ Kg Ans

(f) Angular position of the balancing mass -

$\theta_A = 200^\circ$ Ans

(g) Fluctuation in rail pressure -

$\because$ each balancing mass = 105 Kg

$\therefore$ Balancing mass for rotating mass -

$= \frac{m_1}{m_2} \times 105$

$= \frac{150}{270} \times 105$

$= 58.3$ Kg

(h) Tractive effort -

$= \pm \sqrt{2}\,(1-c)\, m_2 \cdot \omega^2 \cdot r$

$= \pm \sqrt{2}\left(1-\frac{2}{3}\right) 180\,(31.42)^2\, 0.314$

$= \pm 25127$ N Ans