Example 1.8.1 : *Derive the expression for signal to quantization noise ratio for PCM system that employs linear quantization technique. Assume that input to the PCM system is a sinusoidal signal.*

OR

A PCM system uses a uniform quantizer followed by a v bit encoder. Show that rms signal to quantization noise ratio is approximately given by (1.8 + 6v) dB.

Solution : Assume that the modulating signal be a sinusoidal voltage, having peak amplitude A_m. Let this signal cover the complete excursion of representation levels.

The power of this signal will be,

$$P = \frac{V^2}{R} \quad \text{Here V = rms value}$$

$$= \left[A_m / \sqrt{2}\right]^2 \quad \text{... (1.8.39)}$$

When $R = 1$, the power P is normalized, i.e.,

Normalized power : $P = \frac{A_m^2}{2}$ with $R = 1$ in above equation.

∴ Signal to quantization noise ratio is given by equation 1.8.33 as,

$$\frac{S}{N} = \frac{3P}{x_{max}^2} \times 2^{2v}$$

Here $P = \frac{A_m^2}{2}$ and $x_{max} = A_m$

Putting these values in the above equation,

$$\frac{S}{N} = \frac{3 \times \frac{A_m^2}{2}}{A_m^2} \times 2^{2v} = \frac{3}{2} \times 2^{2v} = 1.5 \times 2^{2v}$$

Expressing signal to noise power ratio in dB,

$$\left(\frac{S}{N}\right) dB = 10 \log_{10}\left(\frac{S}{N}\right) = 10 \log_{10}\left(1.5 \times 2^{2v}\right)$$

$$= 10 \log_{10}(1.5) + 10 \log_{10} 2^{2v}$$

$$= 1.76 + 2v \times 10 \times 0.3$$

Thus,

$$\left(\frac{S}{N}\right) dB \text{ in PCM} : \left(\frac{S}{N}\right) dB = 1.8 + 6v \text{ ; for sinusoidal signal}$$

Example 1.8.2 : *A Television signal with a bandwidth of 4.2 MHz is transmitted using binary PCM. The number of quantization levels is 512.*
Calculate,
i) Code word length *ii) Transmission bandwidth*
iii) Final bit rate *iv) Output signal to quantization noise ratio.*

[March-2003, 10 Marks]

Solution : The bandwidth is 4.2 MHz, means highest frequency component will have frequency of 4.2 MHz i.e.,

$$W = 4.2 \text{ MHz}$$

Quantization levels $q = 512$

i) Number of bits and quantization levels are related in binary PCM as,

$$q = 2^v$$

i.e. $$512 = 2^v$$

$$\log 512 = v \log 2$$

or $$v = \frac{\log 512}{\log 2}$$

$$= 9 \text{ bits} \qquad \text{... (Ans)}$$

Thus the code word length is 9 bits.

ii) From equation 1.8.6 the transmission channel bandwidth is given as,

$$B_T \geq vW$$

$$\geq 9 \times 4.2 \times 10^6 \; Hz$$

$$B_T \geq 37.8 \text{ MHz} \qquad \text{... (Ans)}$$

iii) The final bit rate will equal to signaling rate. From equation 1.8.3 signaling rate is given as,

$$r = v f_s$$

Sampling frequency $f_s \geq 2W$ by sampling theorem.

$$\therefore \quad f_s \geq 2 \times 4.2\, MHz \quad \text{since } W = 4.2 \text{ MHz}$$

$$\therefore \quad f_s \geq 8.4 \text{ MHz}$$

Putting this value of 'f_s' in equation for signaling rate,

$$r = 9 \times 8.4 \times 10^6$$

$$= 75.6 \times 10^6 \text{ bits/sec} \qquad \textbf{... (Ans)}$$

From equation 1.8.4 transmission bandwidth is also obtained as,

$$B_T \geq \frac{1}{2} r$$

$$\geq \frac{1}{2} \times 75.6 \times 10^6 \quad \text{bits/sec}$$

or $B_T \geq 37.8$ MHz which is same as the value obtained earlier.

iv) The signal to noise ratio

$$\left(\frac{S}{N}\right) dB \leq 4.8 + 6v\, dB$$

$$\leq 4.8 + 6 \times 9$$

$$\leq 58.8 \text{ dB} \qquad \textbf{... (Ans)}$$

Q. 2 Explain Pulse code modulation.

1.8.1 PCM Generator

The pulse code modulator technique samples the input signal $x(t)$ at frequency $f_s \geq 2W$. This sampled 'Variable amplitude' pulse is then digitized by the analog to digital converter. The parallel bits obtained are converted to a serial bit stream. Fig.1.8.1 shows the PCM generator.

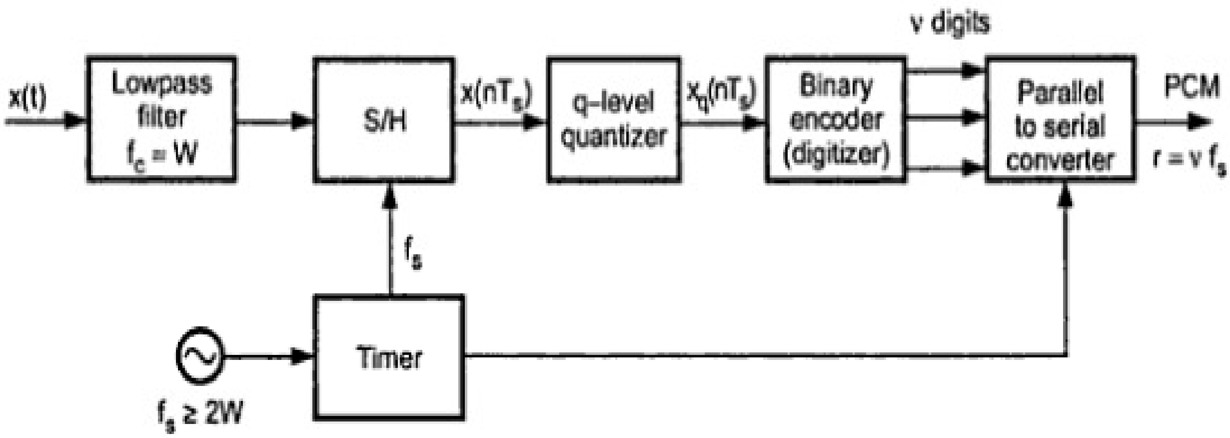


Fig. 1.8.1 PCM generator

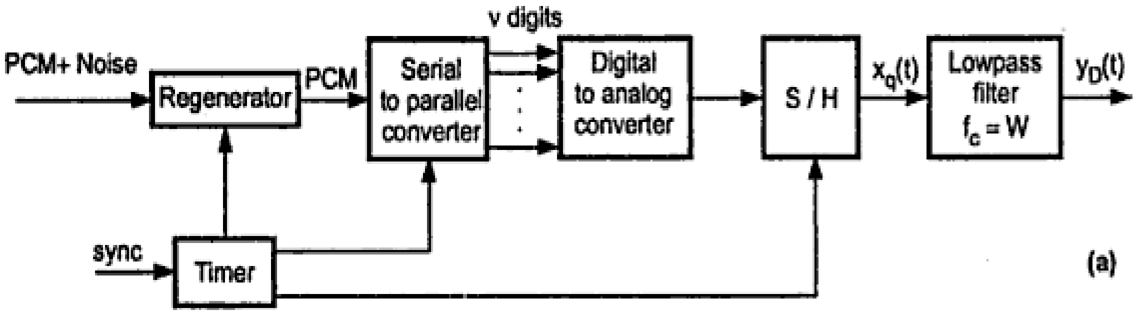
In the PCM generator of above figure, the signal $x(t)$ is first passed through the lowpass filter of cutoff frequency 'W' Hz. This lowpass filter blocks all the frequency components above 'W' Hz. Thus $x(t)$ is bandlimited to 'W' Hz. The sample and hold circuit then samples this signal at the rate of f_s. Sampling frequency f_s is selected sufficiently above Nyquist rate to avoid aliasing i.e.,

$$f_s \geq 2W$$

In Fig. 1.8.1 output of sample and hold is called $x(nT_s)$. This $x(nT_s)$ is discrete in time and continuous in amplitude. A q-level quantizer compares input $x(nT_s)$ with its fixed digital levels. It assigns any one of the digital level to $x(nT_s)$ with its fixed digital levels. It then assigns any one of the digital level to $x(nT_s)$ which results in minimum distortion or error. This error is called *quantization error*. Thus output of quantizer is a digital level called $x_q(nT_s)$.

1.8.3 PCM Receiver

Fig. 1.8.2 (a) shows the block diagram of PCM receiver and Fig. 1.8.2 (b) shows the reconstructed signal. The regenerator at the start of PCM receiver reshapes the pulses and removes the noise. This signal is then converted to parallel digital words for each sample.



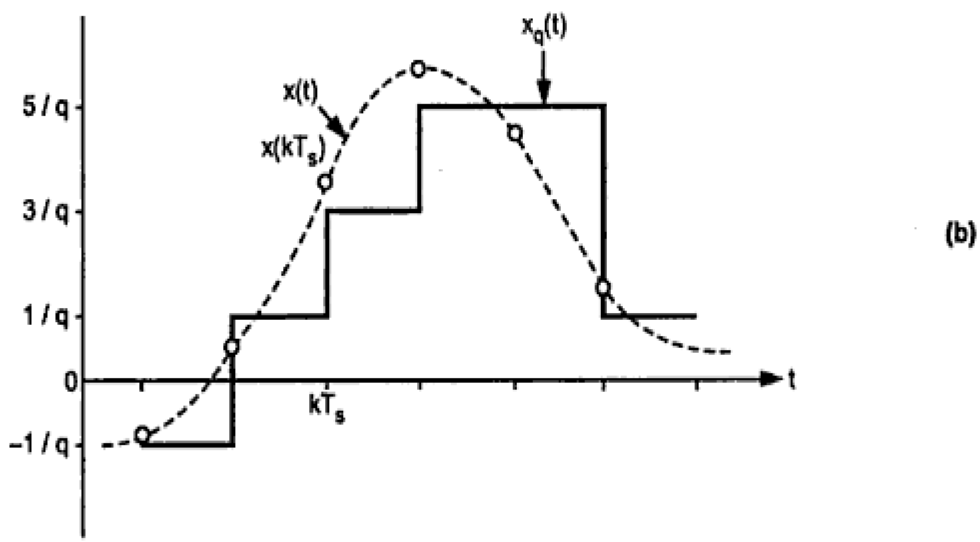


Fig. 1.8.2 (a) PCM receiver
(b) Reconstructed waveform

Q. 3 Explain Linear Quantization.

We know that input sample value is quantized to nearest digital level. This quantization can be uniform or nonuniform. In uniform quantization, the quantization step or difference between two quantization levels remains constant over the complete amplitude range. Depending upon the transfer characteristic there are three types of uniform or linear quantizers as discussed next.

1.8.4.1 Midtread Quantizer

The transfer characteristic of the midtread quantizer is shown in Fig. 1.8.3.

As shown in this figure, when an input is between $-\delta/2$ and $+\delta/2$ then the quantizer output is zero. i.e.,

For $\quad -\delta/2 \le x(nT_s) < \delta/2 \; ; \quad x_q(nT_s) = 0$

Here 'δ' is the step size of the quantizer.

for $\quad \delta/2 \le x(nT_s) < 3\delta/2 \; ; \quad x_q(nT_s) = \delta$

Similarly other levels are assigned. It is called midtread because quantizer output is zero when $x(nT_s)$ is zero. Fig.1.8.3 (b) shows the quantization error of midtread quantizer. Quantization error is given as,

$$\varepsilon = x_q(nT_s) - x(nT_s) \qquad \text{... (1.8.7)}$$

In Fig. 1.8.3 (b) observe that when $x(nT_s) = 0$, $x_q(nT_s) = 0$. Hence quantization error is zero at origin. When $x(nT_s) = \delta/2$, quantizer output is zero just before this level. Hence error is $\delta/2$ near this level. From Fig. 1.8.3 (b) it is clear that,

$$-\delta/2 \le \varepsilon \le \delta/2 \qquad \text{... (1.8.8)}$$

Thus quantization error lies between $-\delta/2$ and $+\delta/2$. And maximum quantization error is, maximum quantization error, $\varepsilon_{max} = \left|\frac{\delta}{2}\right|$ ... (1.8.9)

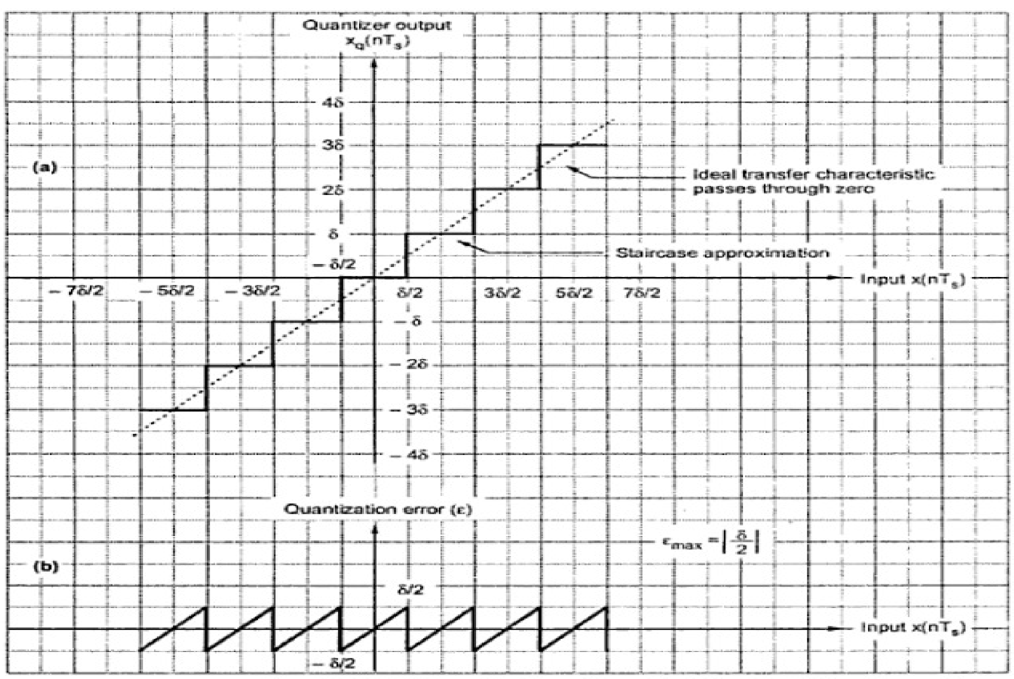


Fig. 1.8.3 (a) Quantization characteristic of midtread quantizer (b) Quantization error

1.8.4.2 Midriser Quantizer

The transfer characteristic of the midriser quantizer is shown in Fig. 1.8.4.

In Fig. 1.8.4 observe that, when an input is between 0 and δ, the output is $\delta/2$. Similarly when an input is between 0 and $-\delta$, the output is $-\delta/2$. i.e.,

For $\quad 0 \le x(nT_s) < \delta \; ; \quad x_q(nT_s) = \delta/2$

$\quad -\delta \le x(nT_s) < 0 \; ; \quad x_q(nT_s) = -\delta/2$

Similarly when an input is between 3 δ and 4 δ, the output is $7\delta/2$. This is called midriser quantizer because its output is either $+\delta/2$ or $-\delta/2$ when input is zero.

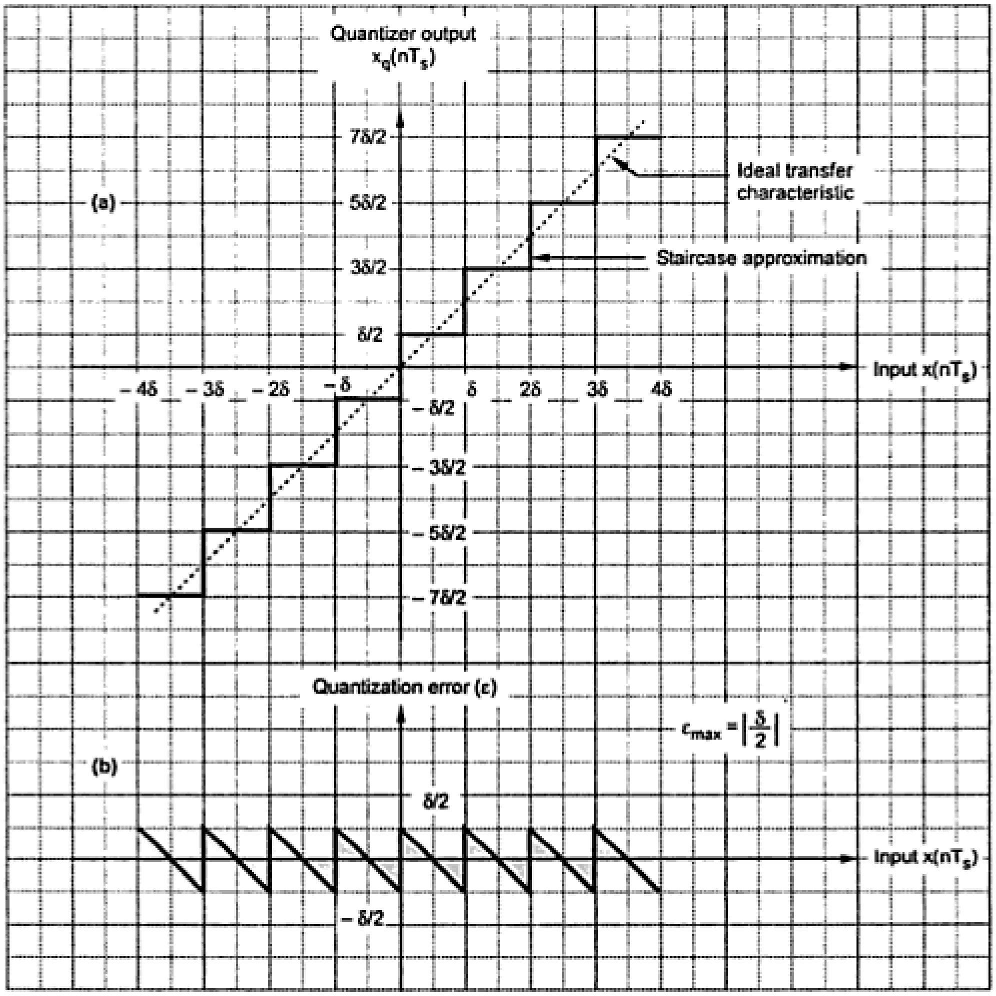


Fig. 1.8.4 (a) Transfer characteristic of midriser quantizer (b) Quantization error

Fig. 1.8.4 (b) shows the quantization error in midriser quantization. When input $x(nT_s) = 0$, the quantizer will assign the level of $\delta/2$. Hence quantization error will be,

$$\varepsilon = x_q(nT_s) - x(nT_s)$$

$$= \delta/2 - 0 = \delta/2$$

Thus the quantization error lies between $-\delta/2$ and $+\delta/2$. i.e.,

$$-\delta/2 \leq \varepsilon \leq \delta/2 \quad \text{... (1.8.10)}$$

And the maximum quantization error is,

$$\varepsilon_{max} = \left|\frac{\delta}{2}\right| \quad \text{... (1.8.11)}$$

Example 1.8.1 : *Derive the expression for signal to quantization noise ratio for PCM system that employs linear quantization technique. Assume that input to the PCM system is a sinusoidal signal.*

OR

A PCM system uses a uniform quantizer followed by a v bit encoder. Show that rms signal to quantization noise ratio is approximately given by (1.8 + 6v) dB.

Solution : Assume that the modulating signal be a sinusoidal voltage, having peak amplitude A_m. Let this signal cover the complete excursion of representation levels.

The power of this signal will be,

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$$= \left[A_m / \sqrt{2}\right]^2 \qquad \text{... (1.8.39)}$$

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Normalized power : $P = \frac{A_m^2}{2}$ with $R = 1$ in above equation.

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$$\frac{S}{N} = \frac{3P}{x_{max}^2} \times 2^{2v}$$

Here $P = \frac{A_m^2}{2}$ and $x_{max} = A_m$

Putting these values in the above equation,

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[March-2003, 10 Marks]

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$$= 9 \text{ bits} \quad \text{... (Ans)}$$

Thus the code word length is 9 bits.

ii) From equation 1.8.6 the transmission channel bandwidth is given as,

$$B_T \geq vW$$

$$\geq 9 \times 4.2 \times 10^6 \ Hz$$

$$B_T \geq 37.8 \text{ MHz} \quad \text{... (Ans)}$$

iii) The final bit rate will equal to signaling rate. From equation 1.8.3 signaling rate is given as,

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Sampling frequency $f_s \geq 2W$ by sampling theorem.

$\therefore \quad f_s \geq 2 \times 4.2 \, MHz$ since $W = 4.2$ MHz

$\therefore \quad f_s \geq 8.4$ MHz

Putting this value of 'f_s' in equation for signaling rate,

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iv) The signal to noise ratio

$$\left(\frac{S}{N}\right) dB \leq 4.8 + 6v \ dB$$

$$\leq 4.8 + 6 \times 9$$

$$\leq 58.8 \text{ dB} \quad \text{... (Ans)}$$

Example 1.8.7 : *A Signal of bandwidth 3.5 kHz is sampled quantized and coded by a PCM system. The coded signal is then transmitted over a transmission channel of supporting a transmission rate of 50 k bits/sec. Calculate the maximum signal to noise ratio that can be obtained by this system.*
The input signal has peak to peak value of 4 volts and rms value of 0.2 V.

Solution : The maximum frequency of the signal is 3.5 kHz,

i.e. $W = 3.5 \text{ kHz}$

Therefore sampling frequency will be

$$f_s \geq 2W$$
$$\geq 2 \times 3.5 \text{ kHz}$$
$$\geq 7 \text{ kHz}$$

The signaling rate is given by equation 1.8.3 as,

$$r = v f_s$$

Putting values of $r = 50 \times 10^3$ bits/sec and $f_s \geq 7 \times 10^3$ Hz in above equation.

$$50 \times 10^3 \leq v \cdot 7 \times 10^3$$

$\therefore$ $v \geq 7.142$ bits

$\therefore$ $v \approx 8$ bits

The rms value of the signal is 0.2 V. Therefore the normalized signal power will be,

$$\text{Normalized signal power} = \frac{(0.2)^2}{1} \quad [R = 1 \text{ for normalized power}]$$

i.e., $P = 0.04 \text{ W}$

The maximum signal to noise ratio is given by

$$\frac{S}{N} = \frac{3P \cdot 2^{2v}}{x_{max}^2}$$

Putting the values of $P = 0.04$, $v = 8$ and $x_{max} = 2$ in above equation,

$$\frac{S}{N} = \frac{3 \times 0.04 \times 2^{2 \times 8}}{4}$$

$$= 1966.08 \approx 33 \, dB$$

Q. Explain Companding in PCM system.

Normally we don't know how the signal level will vary in advance. Therefore the nonuniform quantization (variable step size 'δ') becomes difficult to implement. Therefore the signal is amplified at low signal levels and attenuated at high signal levels. After this process, uniform quantization is used. This is equivalent to more step size at low signal levels and small step size at high signal levels. At the receiver a reverse process is done. That is signal is attenuated at low signal levels and amplified at high signal levels to get original signal. Thus the compression of signal at transmitter and expansion at receiver is called combinely as *companding*. Fig. 1.8.9 shows compression and expansion curves.

As can be seen from Fig. 1.8.9, at the receiver, the signal is expanded exactly opposite to compression curve at transmitter to get original signal. A dotted line in the Fig. 1.8.9 shows uniform quantization. The compression and expansion is obtained by passing the signal through the amplifier having nonlinear transfer characteristic as shown in Fig. 1.8.9. That is nonlinear transfer characteristic means compression and expansion curves.

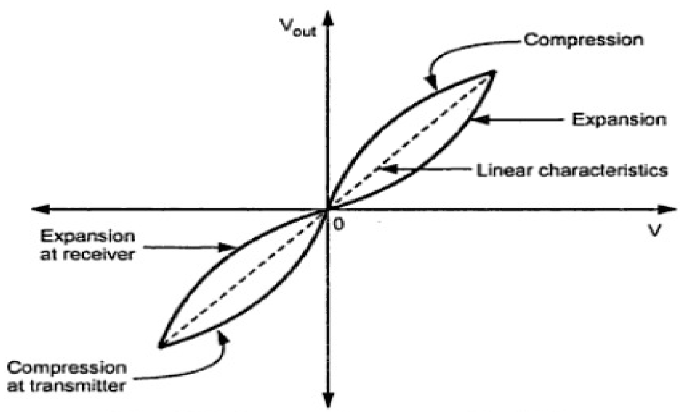

Fig. 1.8.9 Companding curves for PCM

1.8.6.4 μ - Law Companding for Speech Signals

Normally for speech and music signals a μ - law compression is used. This compression is defined by the following equation,

$$Z(x) = (\text{Sgn } x)\frac{ln(1+\mu|x|)}{ln(1+\mu)} \quad |x| \le 1 \qquad \dots (1.8.52)$$

Fig. 1.8.10 shows the variation of signal to noise ratio with respect to signal level without companding and with companding.

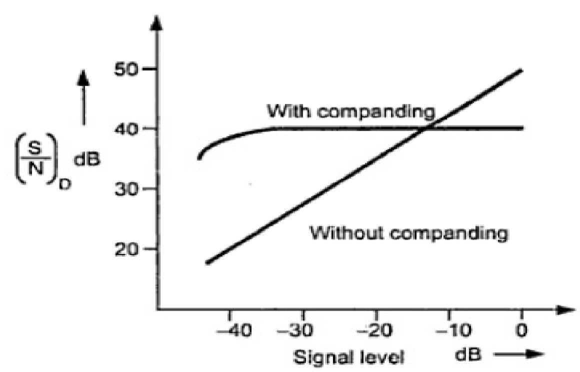

Fig. 1.8.10 PCM performance with μ - law companding

It can be observed from above figure that signal to noise ratio of PCM remains almost constant with companding.

1.8.6.5 A-Law for Companding

The A law provides piecewise compressor characteristic. It has linear segment for low level inputs and logarithmic segment for high level inputs. It is defined as,

$$Z(x) = \begin{cases} \dfrac{A|x|}{1+\ln A} & \text{for } 0 \le |x| \le \dfrac{1}{A} \\ \dfrac{1+\ln(A|x|)}{1+\ln A} & \text{for } \dfrac{1}{A} \le |x| \le 1 \end{cases} \quad \dots (1.8.53)$$

When A = 1, we get uniform quantization. The practical value for A is 87.56. Both A-law and μ-law companding is used for PCM telephone systems.

1.8.6.6 Signal to Noise Ratio of Companded PCM

The signal to noise ratio of companded PCM is given as,

$$\frac{S}{N} = \frac{3q^2}{[\ln(1+\mu)]^2} \quad \dots (1.8.54)$$

Here $q = 2^{\nu}$ is number of quantization levels.

Q. Explain T1 System.

The multiple channel alignment is very important in TDM/PCM system. Fig. 1.9.3 shows the TDM frame format of most widely used T1 system.

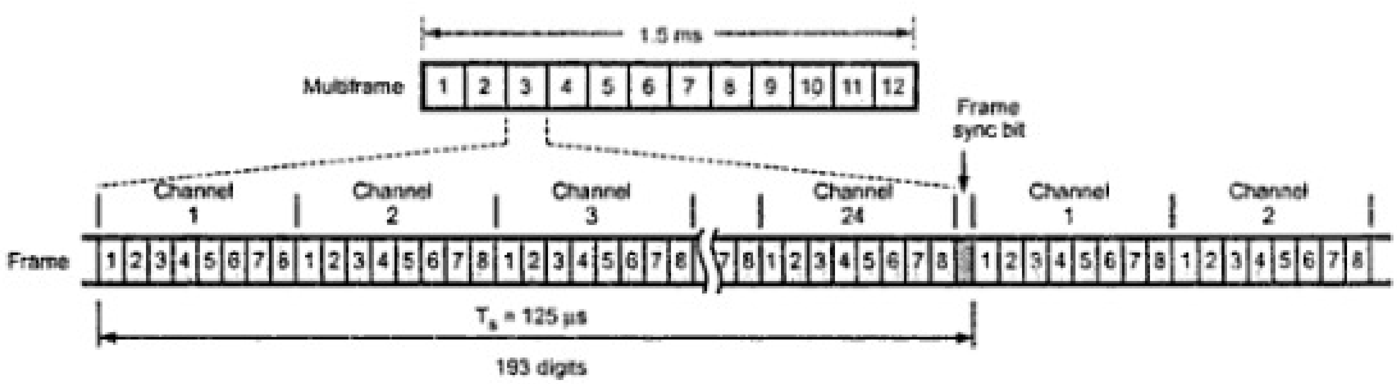

Fig. 1.9.3 Multiple channel frame alignment in T1 system